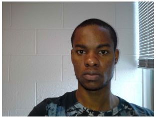
Crystalline topological phases and quantum anomalies

Shinsei Ryu
Univ. of Illinois, Urbana-Champaign

People and papers

Chang-Tse Hsieh (UIUC)

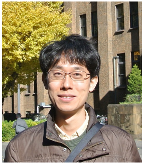


Bode Sule (UIUC)

Xiao Chen (UIUC)

Rob Leigh (UIUC)

Gil Young Cho (UIUC)



Takahiro Morimoto
(RIKEN -> Berkeley)

C.-T. Hsieh, O. M. Sule, G. Y. Cho, SR and R. G. Leigh, PRB (2014)
C.-T. Hsieh, T. Morimoto, SR, PRB (2014)
G. Cho, C.-T. Hsieh, Morimoto, SR, arXiv. (2015)
C.-T. Hsieh, G. Cho, SR, arxiv (2015)

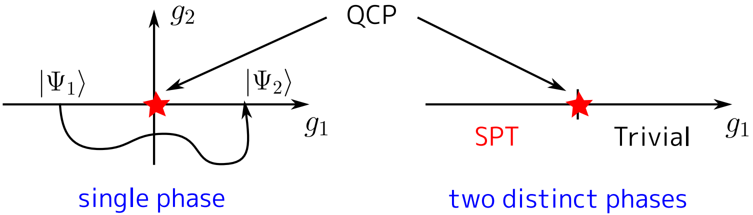
Shou-Cheng Zhang (Stanford)
Hong Yao (Tsinghua)

- Introduction
- Klein bottle partition function
- E.g. 1: Crystalline topological insulator
- Crosscap states
- E.g. 2: Crystalline topological SC and interaction effects
- Summary

Symmetry protected topological phases (SPTs)

- Not topologically-ordered
  "deformable" to a trivial phase (state w/o entanglement)

- But sharply distinct from trivial state once symmetries are enforced:



- E.g. topological insulator

- (often) accompanied by non-chiral edge state
  "unstable" w/o symmetry

Example: 1d Kitaev chain with TRS

Fidkowski-Kitaev

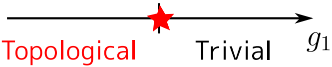
- 1d Kitaev chain w/ TRS (integer classification):

$$H = uH_1 + vH_2$$

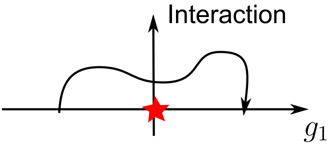
$$H_1 = \sum_j (-c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j^\dagger + h.c.)$$
$$H_2 = \sum_j (c_j^\dagger c_j - 1/2)$$

- Make Nf copies (topological invariant = Nf): $H = \sum_{a=1}^{N_f} (uH_{1a} + vH_{2a})$

- Add interactions to go from topological to trivial:



Two distinct phases ?



Single phase!

- Possible only when Nf = 8 (Nf = 0 mod 8)

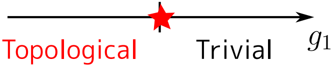
Other known "collapses" of non-interacting classification

- 1d TSC (Kitaev chain w/ TRS) : Z --> Z_8 [Fidkowski-Kitaev (10)]
- 1d TSC with inversion: Z --> Z_4 [Lapa et al. (Taylor's talk)]

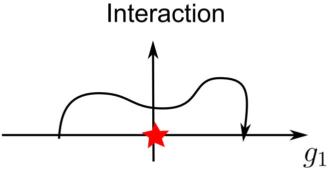
- 2d TSC w/ Z_2 symmetry: Z --> Z_8 [Qi (12), SR-Zhang (12)]
- 2d TSC with reflection symmetry: Z --> Z8 [Yao-SR (12)]

- 3d TSC (3He-B): Z --> Z_{16} [Fidkowski et al (13), Metlitski et al (14), Wang-Senthil (14)]

- 3d crystalline TSC: Z --> Z_{16} [Hsieh-Cho-SR (15)]
- 3d crystalline TI: Z --> Z_8 [Isobe-Fu (15)]
- 3d TSC with inversion: Z-->Z8 [Taylor's talk]



Two distinct phases ?



Single phase!

Motivations

- Is there a general framework to understand all these?

- Can we avoid to go through "microscopic" stability analysis?

  (Microscopic analysis maybe difficult ...)

- Is there a better alternative of non-interacting topological invariants?

- Some progresses for unitary, on-site symmetries.

- How about antiunitary symmetry (time-reveral), spatial (e.g., reflection) symmetries?

Quantum anomalies

Quantum anomalies:
breakdown of a classical symmetry by quantum effects
(nothing is more quantum than this)

Topological phases:
no analogous phase in classical systems (very quantum)

- A close relation known as bulk-boundary correspondence

- Advantage:
  - Robust against interactions, e.g., Adler-Bardeen's theorem
  - Observable: anomaly = "response"
    Operational definition of topological phases

Three ways of symmetry breaking in nature:
Explicit, Spontaneous (Landau Theory), Anomalous (SPT phases)

SPT phases protected by spatial symmetries

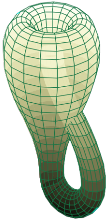
- Non-spatial v.s. spatial symmetries.

  This talk: spatial symmetries

  E.g., Parity symmetry.
  Topological crystalline insulators
  Topological crystalline superconductor

- Is there an anomaly characterizing crystalline topological insulators and superconductors ?
Are they stable agaist interactions?

- Proposed scheme --> "Orientifold" field theory
  (Edge) theories defined on non-orientalble space-time

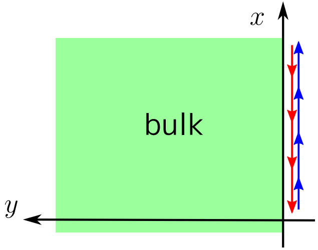


E.g. 1: CP symmetric topological insulator

- System with CP and charge U(1) symmetries

$$P : (x, y) \to (-x, y)$$

"CPT-dual" of QSHE



- Edge Hamiltonian:

$$H = \int dx \left[\psi_L^\dagger i\partial_x \psi_L - \psi_R^\dagger i\partial_x \psi_R \right]$$

- CP symmetry

$$\mathcal{U}\psi_L(x)\mathcal{U}^{-1} = \psi_R^\dagger(-x)$$
$$\mathcal{U}\psi_R(x)\mathcal{U}^{-1} = \psi_L^\dagger(-x)$$

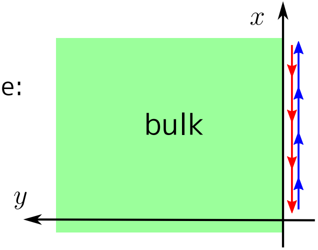
- Can check no mass terms are allowed when topological.
  Z2 classification

- How about interactions ?

E.g.2: Topological crystalline superconductors

- Topological superconductor protected by parity (P)

$$P : (x, y) \to (-x, y)$$



- BdG Hamiltonian on reflection symmetric edge:

$$H = \int dx \, [\psi_L(-i\partial_x)\psi_L + \psi_R(i\partial_x)\psi_R]$$

- TR symmetry

$$T\psi_L(x)T^{-1} = \psi_R(x)$$
$$T\psi_L(x)T^{-1} = -\psi_R(x)$$

- Reflection

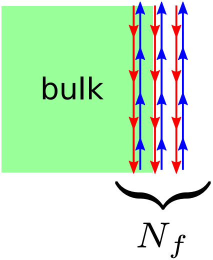
$$P\psi_R(x)P^{-1} = \psi_L(-x)$$
$$P\psi_L(x)P^{-1} = \psi_R(-x)$$

- Classification is Z (protected by Mirror Chern number)

Collapse of non-interacting classification Z --> Z_8

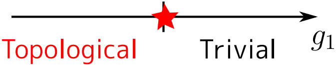
H. Yao and SR (12)

- 2d SCs with T-symmetry and reflection:
  Classified by an integer topological invariant.

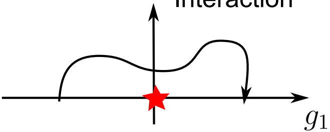


- Edge states are stable at non-interacting level.

- Consider N_f copies of Crystalline TSCs.
  By adding interactions,
  the edge state is unstable when N_f = 8.



Two distinct phases ?



Single phase!

Anomalous symmetry breaking

- Starting point: Edge/surface theory with reflection

E.g.) $$H = \int dx \sum_{a=1}^{N_f} [\psi_L^a (-iv\partial_x)\psi_L^a + \psi_R^a (iv\partial_x)\psi_R^a]$$

$$P\psi_R(x)P^{-1} = \psi_L(-x)$$
$$P\psi_L(x)P^{-1} = \psi_R(-x)$$

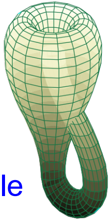
- Step1: Enforcing reflection symmetry by putting the edge theory on a unoriented surface, the Klein bottle.

Can be acheived by twisted boundary conditions

Twisting by reflection

$$\psi_L(t+T,x) = \psi_R(t,\ell - x)$$
$$\psi_R(t+T,x) = \psi_L(t,\ell - x)$$

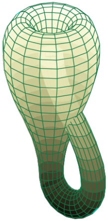
Klein bottle



Anomalous symmetry breaking

- Step2: Study the effects of other symmetires (U(1) or TRS)
  on the Klein bottle

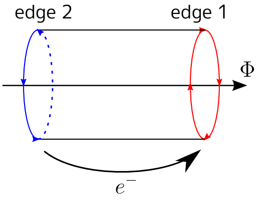
  Criterion:
  Broken symmetry --> topological phase
  Unbroken symmetry --> trivial phase

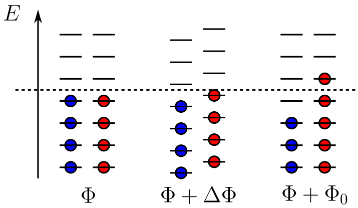


- Comment 1:
  A proper generalization of Laughlin's charge pumping argument.

- Comment 2:
  Subtle form of symmetry breaking by quantum effects: "**quantum anomaly**"

Laughlin's gauge argument = gauge anomaly





- Adiabatic process $\Phi \to \Phi + \Delta\Phi$

- When $\Delta\Phi = \mathrm{integer} \times \Phi_0$ system goes back to itself
  ("large gauge equivalent")
$$H(\Phi) \equiv H(\Phi + n\Phi_0)$$

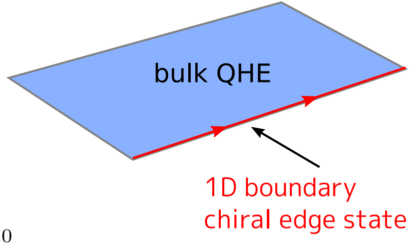
- However, by this adiabatic process, an integer multiple of charge is transported from the left (right) to right (left) edge.

- Charge is not conserved for a given edge.

Laughlin's argument: edge theory point of view

- Chiral edge theory

$$\mathcal{L} = \frac{1}{2\pi}\psi^\dagger i(\partial_t + v\partial_x)\psi$$



- Boundary conditions

$$\psi(t, x+L) = e^{2\pi i a}\psi(t,x), \quad a := \Phi/\Phi_0$$

$$\psi(t+\beta, x) = e^{2\pi i b}\psi(t,x). \quad \text{b: chemical potential}$$

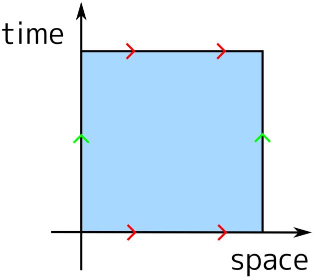
- Classical system (Lagrangian + BCs) is invariant under a -> a+1 and b-> b+1

- Quantum mechanics --> Large gauge anomaly

$$Z_{[a,b]} := \int \mathcal{D}[\psi^\dagger, \psi] e^{-S}$$

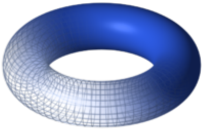
$$Z_{[a,b]} \neq Z_{[a,b+1]} \qquad Z_{[a,b]} \neq Z_{[a+1,b]}$$

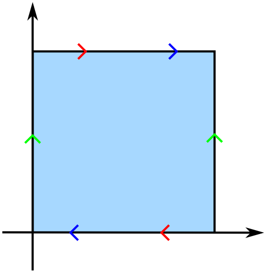
Twisting boundary conditions



Twisting by on-site symmetry

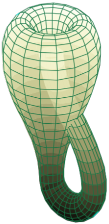
$$Z = \mathrm{Tr}_h \left[g e^{-\beta H} \right]$$
$$\Phi(t+T, x) = g \cdot \Phi(t, x)$$
$$\Phi(t, x+L) = h \cdot \Phi(t, x)$$





Twisting by parity symmetry

$$Z = \mathrm{Tr}_h \left[P\, e^{-\beta H} \right]$$
$$\Phi(t+T, x) = g \cdot \Phi(t, L-x)$$
$$\Phi(t, x+L) = h \cdot \Phi(t, x)$$



Simple example: CP symmetric TI

- Edge Hamiltonian:

$$H = \int dx \left[\psi_L^\dagger i\partial_x \psi_L - \psi_R^\dagger i\partial_x \psi_R \right]$$

$$\mathcal{U}\psi_L(x)\mathcal{U}^{-1} = \psi_R^\dagger(-x)$$
$$\mathcal{U}\psi_R(x)\mathcal{U}^{-1} = \psi_L^\dagger(-x)$$

- Klein bottle partition function:

$$\psi_L(t+T,x) = \psi_R^\dagger(t,\ell - x)$$
$$\psi_R(t+T,x) = \psi_L^\dagger(t,\ell - x)$$

Twisting by CP

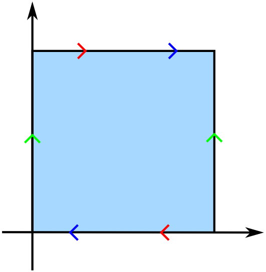
$$\begin{aligned} \psi_L(x+\ell) &= e^{2\pi i a}\psi_L(x), \\ \psi_R(x+\ell) &= e^{2\pi i a}\psi_R(x). \end{aligned}$$

Twisting by U(1)

- Classically invariant under a-> a+1

- But not quantum mechanically:

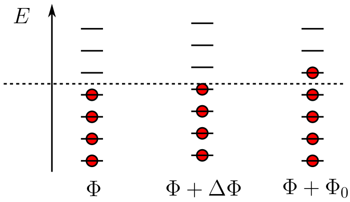
$$Z^{\text{Klein}}_{[a+1]} = -Z^{\text{Klein}}_{[a]}$$



Laughlin's argument revisted

- Chiral edge theory

$$\mathcal{L} = \psi^\dagger i(\partial_t + v\partial_x)\psi$$



- Twisted boundary condition

$$\psi(x) - e^{2\pi i a}\psi(x+\ell) = 0$$

- Ground state with twisted BC: $|GS\rangle_a$

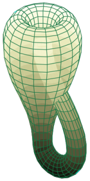
$$\left[\psi(x) - e^{2\pi i a}\psi(x+\ell)\right]|GS\rangle_a = 0$$

- Ground state charge is not invariant
  (Q = U(1) charge)

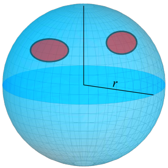
$$e^{i2\pi bQ}|GS\rangle_a = e^{2\pi iba}|GS\rangle_a$$

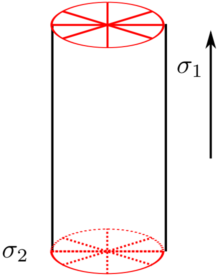
Reformulation using crosscap states

- Klein bottle = sphere with two crosscaps



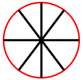
$$\simeq$$





- Finding a nice time slice --> "Crosscap" state |C>:

$$\left[\Phi(0,\sigma_2+\beta) - U\cdot\Phi(0,\sigma_2)\right]|C\rangle = 0$$

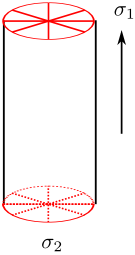


circumference: 2β

c.f. twisted BC

$$\left[\Phi(x+\ell,\tau) - U_G\cdot\Phi(x,\tau)\right]|\sigma\rangle = 0$$

Anomalous crosscap states



- Symmetry G acting on crosscap [e.g. G= U(1)]

$$\mathcal{G}\Phi(\sigma_2)\mathcal{G}^{-1} = U_G\Phi(\sigma_2)$$

$$\mathcal{G}\ [\Phi(\sigma_2+\beta) - U\cdot\Phi(\sigma_2)]\ \mathcal{G}^{-1}\,\mathcal{G}\,|C\rangle = 0$$

$$\left[\Phi(\sigma_2+\beta) - U_G^{-1}\cdot U\cdot U_G\cdot\Phi(\sigma_2)\right]\ \mathcal{G}\,|C\rangle = 0$$

- When U and U_G commute, crosscap condition is invariant

$$[\Phi(\sigma_2+\beta) - U\cdot\Phi(\sigma_2)]$$

but crosscap state may not be!

$$\mathcal{G}|C\rangle = e^{i\alpha}|C\rangle$$

- Related to the anomalous phase of the partition function

- Z_8 classification of reflection symmetric TSC $\quad T|C\rangle = e^{i2\pi N_f/8}|C\rangle$

Summary

- Formulated Laughlin's argument for SPTs protected by parity and other symmetries

  Topology change from Torus to Klein

- Symmetry properties of crosscap states
  Reproduced Z_2 and Z_8 classifications
  Reproduced classification of bosonic SPTs
  Generic K-matrix theory

- SPT phases (= distinction of phases respecting the same symmetries)
  are not described by Landau theory.
  However, they may be characterized by quantum anomalies.

- Similar analysis in (3+1)d Z_{16} classification [Hsieh-Cho-Ryu (15)]
  Similar analysis in (1+1)d Z_8 classification